QUARK MODELS, UNIVERSALITY, SYMMETRY, AND HIGH-ENERGY SCATTERING

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New relations between meson-baryon and baryon-baryon cross sections are found by making use of predictions for scattering on deuteron targets. These are in better agreement with experiment than previously derived relations and provide a better parametrization for the quark-model amplitudes. They show that universal coupling of ω and φ Regge trajectories to mesons and baryons is a good approximation, but that the ratio of the two couplings is about 20% greater than predicted by SU(3) and SU(6).

Regularities observed in the experimental values of the total cross sections of mesons, protons, and antiprotons on nucleons have been interpreted with remarkable success by quark models[1-4] and by Regge-pole models.[5-8] We present two new relations which are in somewhat better agreement with experiment:

$$\sigma(\bar{p}d)-\sigma(pd)=3[\sigma(K^-d)-\sigma(K^+d)], \tag{1a}$$

$$\sigma(\bar{p}p)-\sigma(pp)=2[\sigma(K^-p)-\sigma(K^+p)] + \sigma(K^-n)-\sigma(K^+n). \tag{1b}$$

The theoretical interpretation of these relations has interesting implications both in the quark model and in the Regge picture. Both (1a) and (1b) are simple linear combinations of relations previously obtained either from the quark model or from Regge exchange with certain symmetry assumptions. However, as shown in Table I, the agreement with experiment[9] for these relations is considerably better than that for Freund's relation[5]

$$\sigma(\bar{p}p)-\sigma(pp)=5[\sigma(\pi^-p-\sigma(\pi^+p)]. \tag{2}$$

The difference between (1b) and (2) seems surprising because they can be obtained from one another by the Johnson-Treiman[10,11] relations which are known to be in reasonable agreement with experiment.

A consistent picture is obtainable by the following observations. All processes in (1a) have a deuteron target and can have only isosinglet-exchange contributions in the t channel.[12] Equation (1b) involves both isosinglet and isotriplet exchange, but has been constructed to hold separately for each contribution. Thus the better relations (1a) and (1b) do not depend upon any relation between isosinglet and isotriplet amplitudes.

In the Regge-pole model, one immediately concludes that universality is a better assumption than symmetry. Relation (1a) follows only from a universal coupling of the ω trajectory to mesons and baryons. Equation (1b) follows only from independent universal couplings of the ω and ρ trajectories to mesons and baryons. Freund's relation (2) involves an additional assumption relating the ω and ρ couplings by U(6, 6) symmetry.

Relation (1a) also indicates the nature of SU(6) and SU(3) symmetry breaking in the isosinglet trajectories. It requires the same mixing in octet and singlet trajectories as in the physical ω and φ mesons. The ω is coupled only to

Table I. Tests of relations between baryon-baryon and meson-baryon cross sections.

Beam momentum (BeV/c)	$\sigma(\bar{p}d)-\sigma(pd)$	$3[\sigma(K^-d)-\sigma(K^+d)]$	$\sigma(\bar{p}p)-\sigma(pp)$	$5[\sigma(\pi^-p)-\sigma(\pi^+p)]$	$2\sigma(K^-p)-\sigma(K^+p)$ $+\sigma(K^-n)-\sigma(K^+n)$
6	29.5 ± 1.8	32.1 ± 1.3	18.7 ± 1.3	11.5 ± 1.8	18.4 ± 0.7
8	26.5 ± 1.8	23.4 ± 1.3	16.4 ± 1.0	12.0 ± 1.8	14.7 ± 0.7
10	...	23.1 ± 1.3	...	...	...
12	21.7 ± 1.8	19.2 ± 1.3	12.3 ± 1.0	8.5 ± 1.8	11.2 ± 0.7
14	21.0 ± 1.8	18.9 ± 1.3	11.8 ± 1.1	7.5 ± 1.8	10.8 ± 0.7
16	19.5 ± 2.1	20.1 ± 1.5	10.5 ± 1.0	6.8 ± 1.8	11.5 ± 0.9
18	...	...	11.6 ± 3.6	7.5 ± 1.8	10.5 ± 1.6

nonstrange quarks; the φ only to strange quarks and not to the nucleon. The ω coupling is obtained by counting the number of nonstrange quarks to give 3 for the proton and 1 for the K meson. In the limit of SU(6) symmetry, where the ω and φ are degenerate, there is no difference between the use of the SU(3) eigenstates and the mixtures corresponding to the physical ω and φ. However, when SU(6) is broken, as it must be, one must choose the physical ω and physical φ mixtures in order to obtain the relation (1a).

This ω-φ mixing has further implications if one wishes to incorporate exchange degeneracy[7,8,13] between even- and odd-signature trajectories. The degeneracy is required for the ρ and A_2 trajectories in order to obtain the observed equality of the K^+p and K^+n cross sections, and suggests that similar mixing should occur in the even- and odd-signature trajectories. This point of view is further supported by the experimental observation that the f^* seems to be decoupled from pions by analogy with the φ. However, such a mixing leads to contradictions with experiment if only a single even-signature nonet is used.[6,14,15] One needs an additional singlet Pomeranchuk trajectory which is not included in the nonet. The only other alternative involves degeneracy of the isotriplet members of the even- and odd-signature nonets and different mixing for the isosinglet members, as has been pointed out by Daboul.[15]

In the quark model, relations (1) and (2) were obtained with the additional assumption that the quark-quark and quark-antiquark scattering amplitudes satisfy the Pomeranchuk theorem except for an isosinglet annihilation channel.[4] Further examination reveals that a nonvanishing isotriplet annihilation amplitude would not affect the derivation of Eq. (1) but would change the relation (2). Modifying the assumptions used in Ref. 4 so as to include a nonvanishing isotriplet annihilation amplitude leads to

$$(\bar{\mathcal{P}}\mathcal{P})-(\mathcal{P}\mathcal{P})=(\bar{\mathfrak{N}}\mathfrak{N})-(\mathfrak{N}\mathfrak{N})=A, \tag{3a}$$

$$(\bar{\mathfrak{N}}\mathcal{P})-(\mathfrak{N}\mathcal{P})=(\bar{\mathcal{P}}\mathfrak{N})-(\mathcal{P}\mathfrak{N})=A', \tag{3b}$$

$$(\bar{\lambda}\mathcal{P})-(\lambda\mathcal{P})=(\bar{\lambda}\mathfrak{N})-(\lambda\mathfrak{N})=0, \tag{3c}$$

where the quarks are denoted by $\mathcal{P}$, $\mathfrak{N}$, and λ and (qq) denotes the imaginary part of the forward qq scattering amplitude. The quantities A and A' are parameters; in Ref. 4, we had $A'=0$. The assumption (3c) on the individual quark amplitudes is sufficient to give the relation (1a). Assumptions (3a) and (3b) are sufficient to give the relation (1b). The assumptions (3) give the Freund relation (2) only when $A'=0$.[16]

Further insight is obtained by examining the following linear combinations of the Johnson-Treiman relations:

$$\sigma(\pi^-p)-\sigma(\pi^+p) = [\sigma(K^-p)-\sigma(K^+p)]-[\sigma(K^-n)-\sigma(K^+n)], \tag{4a}$$

$$\sigma(\pi^-p)-\sigma(\pi^+p) = \tfrac{1}{3}[\sigma(K^-p)-\sigma(K^+p)+\sigma(K^-n)-\sigma(K^+n)]. \tag{4b}$$

Relation (4a) involves only the isotriplet exchange channel. It follows in the Regge-pole model only from the assumption that the ρ trajectory is universally coupled to the isospin current.[17] It follows in the quark model from the assumptions (3) with no restriction on the values of A and A'. Relation (4b) is chosen so that the left-hand side contains a contribution only from the ρ trajectory and the right-hand side only from the ω trajectory. It is obtained in the above derivation if the ω and ρ couplings are related as in SU(6) or if A' in the quark-model derivation is equal to zero.[18]

Table II shows[9] that (4a) is considerably better than (4b) as is well known.[2,4,11] The values of the discrepancies give quantitative estimates of the deviations from universality and symmetry. For the coupling of the π and K mesons to the ρ trajectory, the deviation from universality is less than 10% and is within the experimental error. On the other hand, the ω contribution to Eq. (4b) is about 40% larger than the value predicted by SU(6) from the ρ coupling.

These results suggest that total cross-section differences of mesons and baryons are well described by universal couplings of vector meson trajectories to conserved currents, but that the introduction of such higher symmetries as SU(3) and SU(6) is misleading. The relevant conserved currents, in addition to isospin, are not the hypercharge and baryon currents as originally proposed,[19] which have simple SU(3) transformation properties. Instead they are the "Okubo nonet" linear combinations[20] which might be called the "nonstrange-quark current" and "strange-quark current" coupled, respectively, to the ω and φ, with universal couplings to all hadrons proportional to the number of nonstrange and strange quarks. Furthermore,

Table II. Tests of the Johnson-Treiman relations.

Beam momentum (BeV/c)	$\sigma(\pi^-p)-\sigma(\pi^+p)$	$[\sigma(K^-p)-\sigma(K^+p)]$ $-[\sigma(K^-n)-\sigma(K^+n)]$	$\frac{1}{3}[\sigma(K^-p)-\sigma(K^+p)$ $\pm\sigma(K^-n)-\sigma(K^+n)]$
6	2.3 ± 0.4	2.6 ± 0.6	3.8 ± 0.2
8	2.4 ± 0.4	4.2 ± 0.6	2.8 ± 0.2
10	1.7 ± 0.4	2.1 ± 0.6	2.8 ± 0.2
12	1.7 ± 0.4	1.7 ± 0.6	2.3 ± 0.2
14	1.5 ± 0.4	1.5 ± 0.6	2.2 ± 0.2
16	1.7 ± 0.4	1.4 ± 0.8	2.4 ± 0.3
18	1.5 ± 0.4	1.2 ± 1.4	2.2 ± 0.5

the ratio of the ω and ρ couplings deviates by 20% from symmetry predictions.[21]

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[9]The experimental data used in the preparation of Tables I and II are taken from W. Galbraith, B. W. Jenkins, T. F. Kycia, B. A. Leontić, R. H. Phillips, A. L. Read, and R. Rubinstein, Phys. Rev. 138, B913 (1965).

[10]K. Johnson and S. B. Treiman, Phys. Rev. Letters 14, 189 (1965).

[11]V. Barger and M. H. Rubin, Phys. Rev. 140, B1365 (1965).

[12]Relations involving deuteron targets also deal directly with experimentally measured quantities and are free from errors introduced in extracting neutron cross sections from deuteron data. The striking agreement with experiment shown for Eq. (1a) in Table I would not be as clear in the equivalent relation on nucleon targets, because the experimental errors (using the data from the same experiment) would be 2-3 times as large. Another relation, $\sigma(pd)+\sigma(\bar{p}d)=3\sigma(\pi d)$, is just the familiar $\frac{3}{2}$ ratio between pion-nucleon and nucleon-nucleon cross sections.[1-5] This is already known to be in rough agreement with experiment but with a discrepancy of 10-20%, well outside experimental errors.

One can question the validity of the additivity assumption for quark amplitudes when applied to the deuteron. Note that multiple-scattering and shadow effects on a single quark scattered by a deuteron do not affect the validity of the relation (1a), which assumes only additivity of quark-deuteron amplitudes and does not break down the deuteron into its components. The only multiple effects that lead to a violation of (1a) are those for which correlations between two quarks in the projectile prevent each one from being considered as scattered independently by the deuteron. The agreement of (1a) with experiment indicates that these effects are still small for the deuteron. To see when these multiple effects become important, it might be of interest to check relations such as (1a) with complex nuclei as targets. So long as multiple effects are negligible, all relations such as (1a) that hold for a deuteron target should also hold for any $I=0$ target, e.g., C^{12}, while all relations involving nucleon targets should hold as well for any isodoublet target, e.g., H^3 and He^3.

[13]R. C. Arnold, Phys. Rev. Letters 14, 657 (1965).

[14]See, for example, B. R. Desai and P. G. O. Freund, Phys. Rev. Letters 16, 622 (1966).

[15]J. Daboul, "Compatibility of Quark and Regge-Pole Models," to be published.

[16]The quark-model assumptions (3) and the t-channel quantum numbers are related as follows. The sums and differences of Eqs. (3a) and (3b) relate the t-channel isotriplet and isosinglet amplitudes, respectively. One is proportional to $A-A'$, the other to $A+A'$. The assumption that the two amplitudes are equal is thus equivalent to the assumption in Ref. 4 that $A'=0$.

[17]Relation (4a) depends only upon the ratio of the couplings of pions and kaons to exchanged isotriplet objects which are odd under charge conjugation. Symmetry assumptions that place the exchanged objects in multiplets and relate couplings of different members of these multiplets are completely irrelevant.

[18]J. J. J. Kokkedee [Phys. Letters 22, 88 (1966)] has shown that both Johnson-Treiman relations are obtained if the right-hand sides of our Eqs. (3b) and (3c)

are both equal and not necessarily zero. This possibility is now excluded by the success of the new relation (1a) which depends upon the vanishing of the right-hand side of (3c), and which agrees with experiment better than (4b).

[19]J. J. Sakurai, Ann. Phys. (N.Y.) 11, 1 (1960).

[20]S. Okubo, Phys. Letters 5, 165 (1963).

[21]Note that a universal SU(6) symmetry breaking of 20% means a 20% breaking of SU(3) in the meson couplings that are uniquely determined by SU(3). It therefore does not seem consistent to interpret the SU(6) symmetry breaking in the baryon couplings as a modified d/f ratio which preserves SU(3). That SU(3) symmetry-breaking effects of the order of 20% must always be taken into account is also indicated by the analysis of Ref. 6, which requires SU(3) breaking in the coupling of the Pomeranchuk trajectory to pions and kaons.

ERRATA

MAXIMUM LOSSLESS CURRENT IN A SUPERCONDUCTING FOIL WITH A SURFACE SHEATH. H. J. Fink [Phys. Rev. Letters 17, 696 (1966)].

Equation (4) should read

$$\beta=\int_0^b \Psi^4(y)dy\left[\int_0^b \Psi^2(y)dy\right]^{-2}.$$

Page 698, first column, end of second sentence should read as follows: "$\cdots$ or in part in Refs. 3 and 5 and Fink and Kessinger.[11]"

Page 698, second column, line 2, replace 3.6 by 5.1.

The superconducting wire is thought to be an insert in a very large, normal conducting loop. Its length is assumed to be small compared to the circumference of the loop.

NEW METHOD FOR ASSIGNING BARYON RESONANCES TO SU(3) MULTIPLETS. A. Kernan and W. M. Smart [Phys. Rev. Letters 17, 832 (1966)].

On page 833, Table I, the coefficient in the fifth row, last column should be $[(1-\alpha)\sqrt{32}]/\sqrt{3}$ instead of $-(1-\alpha)\sqrt{32}/\sqrt{3}$. In Table II, the coefficient in the second row, last column should be $(g_8^2\sqrt{512})\alpha(1-2\alpha)$ instead of $(g_8^2\sqrt{512})\alpha(1-1\alpha)$.

The authors thank Dr. G. T. Hoff for informing them that she used this method in Phys. Rev. 139, B671 (1965).